A-Level Physics Revision Notes

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June 2017

1 Particles

$$\text{Specific charge} = \frac{\text{charge}}{\text{mass}}$$

In ${}^A_Z X$ notation: A = mass number (protons + neutrons), Z = proton number.

Isotopes — atoms with same proton number but different mass numbers.

1.1 Stable and unstable nuclei

The strong nuclear force overcomes electrostatic repulsion between protons in the nucleus.

- range: 3-4 fm (about diameter of a small nucleus)
    - attractive from 0.5 fm to around 3 fm or 4 fm, repulsive below 0.5 fm.
- same effect between two protons as two neutrons or a neutron & proton.
- exchange particle: π

1.1.1 Radioactive decay

- Alpha α radiation consists of ${}^4_2\alpha$ particles:

$$^A_Z X \rightarrow ^{A-4}_{Z-2} Y + ^4_2 \alpha$$

- Beta β radiation is fast-moving electrons, hence symbol ${}^{0}_{-1}\beta$ or β^-

$$^A_Z X \rightarrow ^{A}_{Z+1} Y + ^0_{-1} \beta + \overline{V_e}$$

  When a neutron in the nucleus changes into a proton a β^- particle is released and instantly emitted, along with an electron antineutrino.
  The existence of the neutrino was hypothesised to account for the conservation of energy in β^- decay — it went unproven until antineutrinos were detected.

- Gamma γ radiation is electromagnetic radiation emitted by an unstable nucleus with too much energy following α or β emission. It has no mass and no charge.

1.2 Particles, antiparticles and protons

Every particle has a corresponding antiparticle. When antimatter and matter meet, they destroy each other and radiation is released — annihilation.

$$1 \text{ electron Volt} = 1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J}$$

- Annihilation — particle and corresponding antiparticle meet and their mass is converted to radiation energy.
    - 2 photons (γ) are produced to ensure a total momentum of zero following the collision

$$\text{minimum energy of each photon, } hf_{min} = E_0 \text{ where } E_0 \text{ is rest energy of particle}$$

- Pair production — a photon γ creates a particle and corresponding antiparticle.

$$\text{minimum energy of photon needed, } hf_{min} = 2E_0 \text{ where } E_0 \text{ is rest energy of particle}$$

- The antiparticle theory states for every particle there is a corresponding antiparticle that:
  - annihilates the particle & itself if they meet — converting their total mass into photons
  - has exactly the same rest mass as the particle
  - has exactly opposite charge to the particle if the particle is charged.
- The electron's antiparticle is the positron (β^+). Positron emission occurs when a proton changes into a neutron in an unstable nucleus with too many protons.

$$^A_Z X \rightarrow ^A_{Z-1} Y + ^0_{+1} \beta + V_e$$

1.3 Particle interactions

The electromagnetic force between two charged particles or objects is due to the exchange of virtual photons (γ). eg two protons will repel each other.

The weak nuclear force affects only unstable nuclei — it is responsible for neutron → proton (β^-) and proton → neutron (β^+) decay. In both, a particle and antiparticle are created but do not correspond.

- a neutron—neutrino interaction changes the neutron to a proton and results in β^- emission
  - W^- boson exchange particle

$$\mathrm{n} + V_e \rightarrow \mathrm{p} + \beta^-$$

- a proton—antineutrino interaction changes the proton to a neutron and results in β^+ emission.
  - W^+ boson exchange particle

$$\mathrm{p} + \overline{V_e} \rightarrow \mathrm{n} + \beta^+$$

- These interactions are due to the exchange of W bosons. Unlike photons they have:
  - non-zero rest mass
  - very short range $\leq \frac{1}{1000}$ fm
  - positive or negative charge

If no neutrino or antineutrino is present, W^- decays to $\beta^- + \overline{V_e}$, and W^+ decays to $\beta^+ + V_e$. Note that charge is conserved.

- β^- decay: $\mathrm{n} \rightarrow \mathrm{p} + \beta^- + \overline{V_e}$
- β^+ decay: $\mathrm{p} \rightarrow \mathrm{n} + \beta^+ + V_e$

In electron capture a proton in a proton-rich nucleus turns into a neutron through weak force interaction with an inner-shell electron.

$$\mathrm{p} + \mathrm{e}^- \rightarrow \mathrm{n} + V_e$$

The same can happen when a proton & electron collide at very high speed. For an electron with sufficient energy the overall change could occur as W^- exchange from e^- to p.

1.4 Particle classifications

Hadrons are particles/antiparticles which interact through the strong force — protons, neutrons, π-ons and K-ons.

Hadrons can interact through all four interactions. They interact trough the strong force and electromagnetic interaction if charged. Other than the proton, which is stable, hadrons tend to decay through the weak force.

Hadrons are further divided into:

- Baryons — protons & all other hadrons incl. neutrons that decay into protons directly or otherwise
- Mesons — hadrons not including protons in their decay products ie π and K mesons.

Leptons do not interact through the strong force — they interact only through the weak, gravitational and (if charged) electromagnetic interactions.

- Lepton decays
  - $\mathrm{K} \rightarrow \pi$, or $\mu + \overline{V_\mu}$, or $\overline{\mu} + V_\mu$
  - $\pi^\pm \rightarrow \mu + \overline{V_\mu}$ or $\overline{\mu} + V_\mu$
  - $\pi^0 \rightarrow \gamma$ (high energy photons)
  - $\mu \rightarrow \mathrm{e}^- + \overline{V_e}$
  - $\overline{\mu} \rightarrow \mathrm{e}^+ + V$
  - Note that decays always obey conservation rules for energy, momentum & charge.

$$\text{rest energy of products} = \text{total energy before} - \text{kinetic energy of products}$$

1.5 Leptons and quarks

Leptons and antileptons can interact to produce hadrons — this is due to the production of quarks during these events.
An up-quark has charge $+\frac{2}{3}$, a down-quark charge $-\frac{1}{3}$

- In a lepton—hadron interaction a neutrino or antineutrino can change into or from a corresponding charged lepton.
  - $V_e + \mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^-$
  - but even though Q conserved $V_e + \mathrm{n} \nrightarrow \overline{\mathrm{p}} + \mathrm{e}^+$
  - this is because the lepton number must balance
- Muon μ decay: μ changes to V_e and e^- created to conserve charge, and $\overline{V_e}$ to preserve lepton number
  - eg $\mu^- \rightarrow \mathrm{e}^- + \overline{V_e} + V_\mu$
  - But $\mu^- \nrightarrow \mathrm{e}^- + \overline{V_e} + \overline{V_\mu}$ even though charge conserved—because lepton number is not.
  - Muon can change only into a muon neutrino (not antineutrino).
  - Electron can only be created with an electron antineutrino
- Lepton number +1 for any lepton, −1 for any antilepton, 0 for non-lepton

- From smallest to greatest rest mass: e^- ...×200... μ^-, $\pi^{0/\pm}$, $\mathrm{K}^{0/\pm}$ ... p
  - $\mathrm{K} \rightarrow \pi$, $\mu^- + \overline{V_\mu}$, or $\mu^+ + V$
  - $\pi^\pm \rightarrow \mu^+\overline{V_\mu}$, or $\mu^- + V_\mu$
  - $\pi^0 \rightarrow \gamma$ (high energy photons)
  - $\mu \rightarrow \mathrm{e}^- + \overline{V_e}$
  - $\overline{\mu} \rightarrow \mathrm{e}^+ + V_e$

1.5.1 Strangeness

Strange particles are produced through the strong interaction and decay through the weak interaction.

- For strangeness +1, need antistrange quark. For strangeness −1, need strange quark.
- Mesons are hadrons consisting of two quarks—one and antiquark.
  - π^0 = any q—$\overline{\mathrm{q}}$ combination — so can be strange
    - $\pi^+ = \mathrm{u}\overline{\mathrm{d}}$, $\pi^- = \overline{\mathrm{u}}\mathrm{d}$
  - each pair of charged mesons is a particle-antiparticle pair
  - antiparticle of any meson is a q—$\overline{\mathrm{q}}$ pair thus another meson.
  - hence only K are strange
    - $\mathrm{K}^0 = \mathrm{d}\overline{\mathrm{s}}$ (+1 strange), $\overline{K^0} = \overline{\mathrm{d}}\mathrm{s}$ (−1 strange)
    - $\mathrm{K}^+ = \mathrm{u}\overline{\mathrm{s}}$ (+1 strange), $K^- = \overline{\mathrm{u}}\mathrm{s}$ (−1 strange)
- Baryons are also hadrons, but consist of three quarks — all of which are antiquarks in an antibaryon.

  - proton = uud, antiproton = $\overline{\mathrm{u}}\overline{\mathrm{u}}\overline{\mathrm{d}}$
  - neutron = udd
  - The proton is the only stable baryon — a free neutron decays into a proton, releasing an electron and antineutrino (β^- decay)

- Quarks are key to β decay
  - β^- decay — d→u quark (neutron to proton)
  - β^+ decay — u→d quark (proton to neutron)

- When balancing equations note that strangeness is conserved in any strong interaction
  - but in weak interactions strangeness can change by 0, +1 or −1 (because strange particles decay in the weak interaction)

2 The photoelectric effect

- If we shine light with high enough frequency on metals, photoelectrons are released.
  - no photoelectrons emitted if the incident frequency < threshold frequency, f_T
  - rate of electron emission $\propto$ intensity
- The photoelectric effect could not be explained by wave theory as this states:
  - for a certain frequency, energy $\propto$ intensity
  - energy would spread evenly across the wavefront
  - each free e$^-$ would gain some energy
  - gradually each free e$^-$ would gain enough to leave
- No explanation for E_k depending only on f, or for the existence of f_T
- could only be explained by the theory of 'packets' ie photons.

$$E = hf = \frac{hc}{\lambda}$$

$$\text{For e}^- \text{ release, } hf \geq \phi \text{ (work function) so } f_T = \frac{\phi}{h}$$

$$hf = \phi + E_{k\ \mathrm{max}}$$

$$\text{Stopping potential gives max } E_k\text{: } e \times V_s = E_{k\ \mathrm{max}}$$

2.1 Energy levels

- e$^-$ can move down an energy level by photoemission
- 'e × V = E_k carried by an electron accelerated through a 1 V potential difference'
- energy gained by electron = accelerating potential difference
- energy carried by each photon is equal to the difference in energy between the two levels (E_2 = lower energy level):
$$\Delta E = E_2 - E_1 = hf$$
- In excitation electrons move up energy levels if they absorb a photon with sufficient energy to cover the difference.
- If electrons emit photons, they can move down energy levels - de-excitation.
  Energy of the photon emitted = $hf = E_1 - E_2$ (E_2 lower level)
- If an electron is removed from an atom it is ionised — energy of each level in the atom is equal to the energy required to ionise from that level.
  - ground state = 'ionisation energy'

Line spectra are evidence for the transitions between discrete energy levels in atoms. If we look at a tube of glowing gas through a prism we see a spectrum of discrete lines, rather than continuous colours. The pattern of wavelengths is unique to each element. The wavelength is linked to the energy of the photons released when electrons de-excite.

2.2 The fluorescent tube

1. An initial high voltage is applied across mercury vapour. This accelerates free electrons, which ionise some of the mercury atoms, producing more free electrons.
2. Free electrons collide with electrons in other mercury atoms, exciting them to higher levels.
3. When the excited electrons return to ground states they emit UV photons.
4. Phosphor coating on the tube absorbs these particles, exciting its electrons.
5. The excited phosphorous electrons de-excite in steps, emitting lower energy visible photons.

2.3 Wave-particle duality

- Interference and diffraction show light as a wave, but the photoelectric effect shows it as a particle.
- Electron diffraction shows the wave nature of electrons
  - diffraction patterns showed when accelerated electrons in vacuo interact with the spaces in graphite crystal
  - following wave theory, the spread of the lines increased if wavelength increased. Slower electrons = wider spacing.

$$\underline{\text{de Broglie}}\ \lambda = \frac{h}{mv}$$

- A vacuum photocell is a glass tube containing two metal plates — a photocathode and photoanode, when light of frequency $\geq f_T$ of the metal is incident on the photocathode, electrons are emitted from the cathode and are attracted to the anode. A microammeter can measure the photoelectric current, which is proportional to the number of electrons per second that transfer from the cathode to the anode

3 Waves

- A progressive wave carries energy from one place to another without transferring any material.
    - transverse — direction of oscillation is perpendicular to direction of energy transfer
    - longitudinal — oscillation is parallel to energy transfer
- displacement — how far a point on the wave has moved from the undisturbed position
- amplitude — maximum magnitude of displacement
- phase — measurement of the position of a certain point along the wave cycle

$$\text{phase difference in radians} = \frac{2\pi d}{\lambda} \text{ for distance } d \text{ apart}$$

- Polarised waves oscillate in only one direction
    - polarisation can only happen for transverse waves
    - a polarising filter only transmits waves in one plane
- Superposition occurs when two or more waves pass through each other — the displacements due to each wave combine.
    - Principle of superposition: *'when two or more waves cross, the resultant displacement equals the vector sum of the individual displacements'*

3.1 Interference

- Interference can be constructive or destructive — matching displacements are constructive, opposite are destructive. If the crest and trough are not of the same magnitude of displacement the destructive interference will not be total.
- For interference to occur the two waves must be coherent — having *'the same wavelength and frequency, and a fixed phase difference'*
- Interference type depends on path difference
    - Constructive interference: path difference = $n\lambda$
    - Destructive interference: path difference = $(n + \frac{1}{2})\lambda$

3.2 Stationary waves

- stationary wave = *'superposition of two progressive waves with the same frequency/wavelength'*. Unlike progressive waves no energy is transferred by a stationary wave.
    - stationary waves vibrating freely do not transfer any energy to their surroundings
- stationary waves on strings and pipes are similar:
    - at f_0: $l = \frac{\lambda}{2}$, $2f_0$: $l = \lambda$, $3f_0$: $l = \frac{3\lambda}{2}$
      where l is a fixed length of string or open pipe.
    - the distance between adjacent nodes is $\frac{\lambda}{2}$
    - a node is a point of zero displacement, an antinode a point of maximum displacement
- the longer/heavier/looser the string, the lower the resonant frequency
  μ = mass per unit length, T = tension

$$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

3.2.1 Explaining stationary waves

- When in phase the two progressive waves reinforce each other to create a larger wave
- $\frac{1}{4}$ of a cycle later the waves have moved $\frac{\lambda}{4}$ in opposite directions so are now in antiphase & cancel each other out
- After another $\frac{1}{4}$ of a cycle they are back in phase with the same resultant as before, but reversed.

The points of zero displacement remain in constant positions throughout. Between these points (nodes) the stationary wave oscillates.

- Phase difference:
  - 0 / 2π if two particles are between adjacent nodes or separated by an even number of nodes
  - π if separated by an odd number of nodes

In a stationary wave all particles except those at the nodes vibrate at the same frequency. The amplitude varies from zero at nodes to maximum at antinodes (whereas in a progressive wave it would be the same for all particles).

Phase difference between two particles is $n\lambda$ where n is the number of nodes between the particles — for a progressive wave phase difference $= \frac{2\pi d}{\lambda}$

The key condition is that the time taken for a wave to travel along the string and back should be equal to the time taken for a whole number of cycles of the vibrator.

3.3 Young's double slit experiment

- Showed the wave nature of light
- Illuminate two closely spaced slits using a suitable light source (Young used one slit prior to the double slit) — the two slits act as coherent sources of light.
- Alternate bright and dark fringes ('Young's fringes') are seen on a screen placed where the diffracted light from the slits overlaps
  - fringes are evenly spaced and parallel to the double slit
- If the single slit prior to the double slit is too wide then each part of it produces a fringe pattern which is displaced slightly from the pattern due to adjacent parts of the slit — as a result the dark fringes of the double slit pattern become narrower than the bright fringes and the contrast is lost between the dark and light fringes.
- Fringes are formed as a result of interference of light from the two slits — a bright fringe is formed where light from each slit arrives in phase with each other, and a dark fringe results where the light waves are in antiphase.

Fringe separation, w, is the distance from the centre of one fringe to the centre of the other. The formula is only valid if w is much smaller than D.

$$w = \frac{\lambda D}{s}$$

- When measuring w, measure across several dark fringes from the centre of one to the centre of another — centres of dark fringes are easier to identify than those of light. Divide this measurement by the number of fringes measured across.
- Two loudspeakers connected to the same signal generator can be used to demonstrate interference as they are coherent sources of sound waves — can detect points of cancellation & reinforcement by ear, and Young's double slit equation can be used.

With white light, each component colour produces its own fringe pattern and each pattern is centred on the screen at the same position.

- central fringe is white because every colour contributes at the centre of the screen
- inner fringes are tinged with blue on the inside and red on the outside because the red fringes are more spaced out than the blue and the two fringe patterns do not overlap exactly
- outer fringes merge into an indistinct background of white light because where the fringes merge different colours reinforce and therefore overlap.

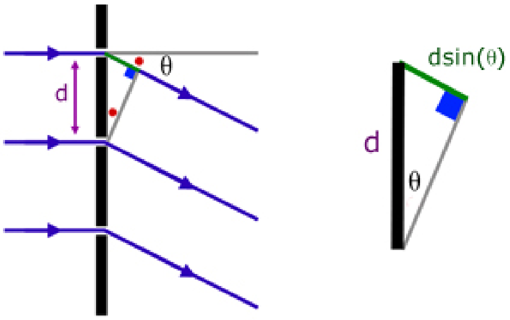

Figure 1: Diffraction grating derivation (a-levelphysicstutor.com)

3.4 Diffraction

- The gap needs to be similar in width to the wavelength. If it is much bigger then there will be no diffraction. Less diffraction occurs when waves pass through a wide gap than a narrow gap.
  - Diffracted waves spread out more if:
    - gap is made narrower, or
    - λ is much larger

3.4.1 Single-slit diffraction

When light passes through a single slit, we get an interference pattern. This consists of a central bright fringe, with dark and bright fringes alternating either side.

- central fringe is twice as wide as other fringes
- outer fringes are all the same width peak intensity decreases with distance from the centre, and outer fringes are much less intense than the central fringe
- greater λ and/or narrower slit = wider fringes

$$\text{width of central fringe} \propto \frac{\lambda}{\text{slit width}}$$

- Thanks to the single-slit effect, Young's (double slit) fringes follow the same intensity distribution as for diffraction through a single slit

3.4.2 Diffraction gratings

- When a parallel beam of monochromatic light is directed normally at a diffraction grating light is transmitted in certain directions only because:
  - light passing through each slit is diffracted
  - diffracted waves from adjacent slits reinforce each other in certain directions only (including the incident light direction) and cancel out in all other directions

$$d \sin\theta = n\lambda \text{ where } d = \text{slit spacing, } n = \text{order (starting at 0)}$$

3.4.3 Deriving the diffraction grating formula

- The wavefront emerging from the slit reinforces a wavefront emitted n cycles earlier from the adjacent slit 'above' it.
- This earlier wavefront must therefore have travelled a distance of $n\lambda$ from the slit — so the distance from the higher slit to the wavefront (the path difference) is $n\lambda$
- θ (angle of diffraction, between beam and zero order) is equal to angle between wavefront and plane of slits so $\sin\theta = \frac{n\lambda}{d}$ where d is the spacing of the two slits

$$d \sin\theta = n\lambda$$

- Increasing λ = fringes more spread out, increasing d = less spread out. $\theta < 90°$ as $\sin 90°$ is the maximum possible.
- X-ray λ similar to the atom spacing in a crystalline structure, so X-rays form a diffraction pattern when directed at thin crystal — the spacing can be found from the diffraction pattern: 'X-ray crystallography'
    - Diffraction gratings are used in spectrometers for studying the spectrum of light from any source and to measure light wavelengths very accurately.
- Maximum number of orders produced is given by $\frac{d}{\lambda}$ rounded down to nearest integer.
- Number of maxima observed is $2n + 1$ where n is the greatest order

3.4.4 Types of spectra

- Continuous
    - most intense part of the spectrum depends on the temperature of the light source — the hotter the light source the shorter the wavelength of the brightest part of the spectrum
- Line emission
    - glowing gas in a vapour lamp or discharge tube emits light at specific wavelengths so its spectrum consists of narrow vertical lines. The wavelengths are characteristic of the element that produced the light
- Line absorption
    - continuous spectrum with dark lines at specific wavelengths. Pattern of dark lines is due to the elements in the glowing gas — these elements absorb light of the same wavelengths they can emit so the transmitted light is missing these wavelengths. The atoms of the gas that absorb light then emit the light subsequently, but not necessarily in the same direction as the transmitted light.

3.5 Refraction

- Absolute refractive index is a measure of optical density

$$n = \frac{c}{c_s}$$

  so the smaller the n of a substance, the greater the speed of light in that substance

- Refractive index between two media, ${}_1n_2$ is a ratio of the speed of light in material 1 to that in material 2

$${}_1n_2 = \frac{c_1}{c_2} = \frac{n_2}{n_1}$$

  We can assume n at an air—substance boundary is the absolute n of a substance.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- When a wave passes from a dense medium into a less dense medium, it bends away from the normal as it speeds up. The reverse is true.

3.5.1 Total internal reflection

- The critical angle is the key to total internal reflection.
    - If light is incident at θ_c to the normal then the ray will exit along the flat surface
    - But if the angle of incidence is greater than θ_c, total internal reflection occurs.
    - Note that TIR will only occur at a boundary of higher n to lower n
- rearranging Snell's law:

$$\sin\theta_c = \frac{n_1}{n_2} =_1 n_2$$

- TIR is useful in fibre optics.

  - The core of the fibre has a high refractive index, but is surrounded by cladding of lower refractive index, which helps to protect the core from scratches (which could allow light to escape) and decreases θ_c to ensure TIR occurs.

- There are several issues encountered with fibre optics
  - Absorption — loss in amplitude as light travels along the fibre. Can be reduced by increasing purity of the glass or using repeaters at frequent intervals.
  - Modal dispersion — light enters the fibre at different angles so can take different paths through the fibre, which results in pulse broadening. Can be mitigated by using monomode fibre.
  - Material dispersion — different wavelengths of light travel at different speeds through the glass (higher n for that λ, lower the speed). Using monochromatic light sources mitigates this issue.

4 Mechanics

4.1 Vectors

- Scalars have magnitude only, whereas vectors have both magnitude and direction.
- Resolving vectors by calculation:
  - Horizontal component: $X = R\cos\theta$
  - Vertical component: $Y = R\sin\theta$
    Note θ measured from the horizontal.
- Finding the resultant vector
  - $\theta = \arctan\frac{Y}{X}$
  - $R = \sqrt{X^2 + Y^2}$
- Free-body diagrams should contain all forces acting on an object but not any forces exerted by the object itself.
- Three coplanar forces acting on a body in equilibrium will form a closed loop — triangle of forces.
- On an inclined plane the weight of the object acts straight down, but the normal reaction at a right angle to the plane. Friction acts against the object sliding down the plane. Note that the angle between mg and the normal to the plane (ie the reaction force) is the same as the slope angle.

4.2 Moments

moment = force × pependicular distance from the line of action of the force to the pivot, unit: N m

The principle of moments says *'for a body to be in equilibrium, the sum of the clockwise moments about any point must equal the sum of the anticlockwise moments about that point'*

- A couple is a pair of coplanar forces of equal size acting parallel to each other but in opposite directions

$$\text{moment of couple} = F \times \text{distance between forces}$$

- The centre of mass of a body is the point through which a single force on the body has no turning effect.
  - If a body is in stable equilibrium, when displaced then released it returns to its original position because c.o.m. is directly below the point of support when the body is at rest.
  - A plank on a drum is in unstable equilibrium — if displaced slightly then released the plank will roll off the drum because the c.o.m. is directly above the point of support when in equilibrium — so weight acts to take it further away from equilibrium position.
  - An object will topple if the line of action of its weight passes beyond the pivot

$$\text{in order for an object to tilt: moment} > mg \times \frac{\text{width of base}}{2}$$

4.3 Motion

- On displacement—time graphs gradient = velocity
- On velocity—time graphs gradient = acceleration & area under graph = displacement

4.3.1 Equations of uniform acceleration

$$v = u + at$$

$$s = \frac{u+v}{2}t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

4.3.2 Projectile motion

- In suvat equations, $a = g$. g always acts downwards ie negative.
- Horizontal and vertical components of motion must be thought of separately
  - vertical motion — constant a due to g being constant
  - horizontal motion — constant speed until projectile lands
- If a projectile is launched at an angle we need to resolve the initial velocity into vertical and horizontal components

$$X = R\cos\theta$$

$$Y = R\sin\theta$$

- resistance $\propto$ speed
- Friction/drag acts in the opposite direction to the motion of the object and converts kinetic energy to heat & sound energy. It increases with speed
- Lift acts perpendicular to fluid flow
- The terminal velocity occurs where the driving force is constant and there is a resistance force which increases with speed
  - Maximum speed is affected by the magnitude of the driving force, and the magnitude of the resistance force.

4.3.3 Newton's Laws of Motion

1. Velocity of an object will not change unless a resultant force acts upon it.
2. Acceleration of an object $\propto$ the magnitude of the resultant force
   - Force is the rate of change of momentum

$$F = ma = m\frac{\Delta V}{\Delta t} = \frac{\Delta(mv)}{t} \text{ therefore } F\Delta t = \Delta(mv)$$

   - The impulse, defined as $F \times t$, thus equals the change in momentum. It is also the area under a force—time graph.
   - all objects fall at the same rate regardless of mass (but resistance does play a part)
3. *'If body A exerts a force on body B, then body B exerts an equal but opposite force on body A'* — aka every action has an equal & opposite reaction.

- Momentum is always conserved, assuming no external forces act.

$$\text{momentum, } p = mv\text{, unit: kg m s}^{-1}$$

  - In an elastic collision, both momentum and energy are conserved.
  - In an inelastic collision, energy is not conserved.

4.4 Energy

$$\text{work done} = \text{force} \times \text{distance moved}$$

Note that force is not always in the same direction as the movement eg for a sled being pulled by a string, only the horizontal force causes the motion.

$$\text{horizontal: } W = Fs\cos\theta$$

$$\text{vertical: } W = Fs\sin\theta$$

- Area under a force—displacement graph tells us the work done

$$P = \frac{\Delta W}{\Delta t} = \frac{E}{t} = Fv$$

4.4.1 Conservation of energy

‘Energy cannot be created nor destroyed, it can be transferred from one form to another, but the total energy in a closed system cannot change’

$$E_k = \frac{1}{2}mv^2$$

$$E_p = mgh$$

5 Materials

$$\text{density, } \rho = \frac{\text{mass, } m}{\text{volume, } v} \text{ unit: kg m}^{-3}$$

- Hooke's law states extension of a stretched object $\propto$ force

$$F = k\Delta l$$

  - This only applies up to the elastic limit, after which the material will be permanently stretched.
  - This plastic deformation results in a non-zero intercept on a F—Δl graph, but the gradient of such a graph remains the same as the forces between bonds are identical.

$$\text{for springs in parallel, effective } k = k_1 + k_2 \ldots + k_n$$
$$\text{for springs in series, } \frac{1}{\text{effective } k} = \frac{1}{k_1} + \frac{1}{k_2} \cdots \frac{1}{k_n}$$

- Elastic — returns to original shape and size when force is removed
- Plastic — material is permanently stretched

5.1 Stress and strain

$$\text{stress} = \frac{F}{A} \text{ unit: Pa, N m}^{-2}$$
$$\text{strain} = \frac{\Delta l}{l} \text{ (no units)}$$

Elastic strain energy is the area below a force—extension graph.

$$\text{Elastic strain energy, } E = \frac{1}{2}F\Delta l = \frac{1}{2}k\Delta l^2$$

The Young modulus is a property of a material — it measures stiffness.

$$\text{Young modulus, } E = \frac{\text{stress}}{\text{strain}} = \frac{Fl}{A\Delta l} \text{ unit: Pa, N m}^{-2}$$

- The gradient of a stress—strain graph is thus equal to the Young modulus.
- Looking at a stress—strain graph, there are three key points: the limit of proportionality, after which the relationship is no longer linear, the elastic limit, past which plastic deformation occurs, and the yield point — after this point, the material suddenly starts to stretch without extra load.
- The stress—strain graph of a brittle material has no curve — it just stops.
- To measure the Young modulus, we need a long thin wire of the material — record the extension and the weight applied, and plot a graph. The graph can then be converted to stress—strain, or as the gradient is $\frac{\Delta l}{F}$, $\frac{1}{\text{gradient}} \times \frac{l}{A}$ = Young modulus

6 Electricity

$$Q = It, \text{ so current=rate of flow of charge}$$

$$W = QV, \text{ so potential difference is the energy per unit charge}$$

$$V = IR, \text{ for an Ohmic conductor } I \propto V. \text{ A shallower gradient on } I\text{—}V \text{ graph = increased resistance}$$

- An Ohmic conductor as a straight line I—V graph
- A silicon diode conducts no current until $V \approx 0.7$ V, after which current flows with very little resistance - the graph should be an almost-vertical line.
- A filament bulb gives an S-curve: greater resistance at higher voltages as the filament heats up due to increased current flow.
- The unknown-resistor circuit consists of a variable resistor in series with the unknown resistance, an ammeter and a voltmeter in parallel with the unknown resistance. It can be used to determine the resistance of the unknown resistor.

6.1 Resistivity

$$\text{resistivity, } \rho l = RA \text{ unit: } \Omega \text{ m}$$

Superconductors have a resistivity of 0 Ω m. These are certain materials, which must be cooled below a 'transition temperature'.
Uses include power transmission lines, strong electromagnets, and very high speed electronic systems.

6.2 Power

$$P = \frac{E}{t} = IV = \frac{V^2}{R} = I^2R$$
$$E = VIt$$

6.3 EMF and internal resistance

The internal resistance of a cell can be imagined much like a resistor in series with the cell.

$$\text{Electromotive force, } \varepsilon = \frac{\text{energy, } E}{Q} = I(R + r) = \text{terminal pd + lost volts}$$

Note that Ohm's Law still applies — $\varepsilon = Ir$
It is helpful to have awareness of potential dividers and resistive input transducers in this section.

7 Circular motion

$$\text{Angular speed, } \omega = \frac{\theta}{t} = 2\pi f \text{ unit: rad s}^{-1}$$

$$\text{Linear velocity, } v = \frac{2\pi r}{T} = 2\pi f r = r\omega$$

$$\text{Magnitude of centripetal accelleration is given by } a = \frac{v^2}{r}$$

$$\text{Using } F = ma,\ F = \frac{mv^2}{r} = mr\omega^2$$

7.1 Humpback bridge & 'looping the loop'

To keep a string taut, the magnitude of the centripetal force must be greater than or equal to the weight.

$$\text{so } mv^2 \geq mg \text{ or alternatively } mr\omega^2 \geq mg$$

$$\text{Tension in the string at the top } = \frac{mv^2}{r} - mg$$

$$\text{Tension in the string at the bottom } = \frac{mv^2}{r} + mg$$

Keeping a car on a humpback bridge requires weight to equal the centripetal force ie $mg \geq \frac{mv^2}{r}$.
Support force from the road $= mg - \frac{mv^2}{r}$.

7.2 Motion round a banked track

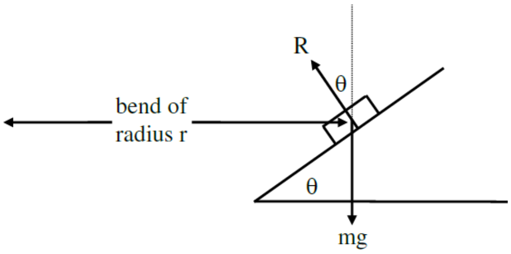

Figure 2: Motion round a banked track

$$\text{The vertical compoment is weight: } mg = R\cos\theta$$

$$\text{The horizontal component is centripetal force: } F = \frac{mv^2}{r} = R\sin\theta$$

$$\text{For there to be no sideways friction: } v = \sqrt{gr\tan\theta}$$

8 Simple harmonic motion

$$\text{Phase difference in radians } = 2\pi\frac{\Delta t}{T}$$

Δt is the time between successive instants where the two objects are at maximum displacement in the same direction.

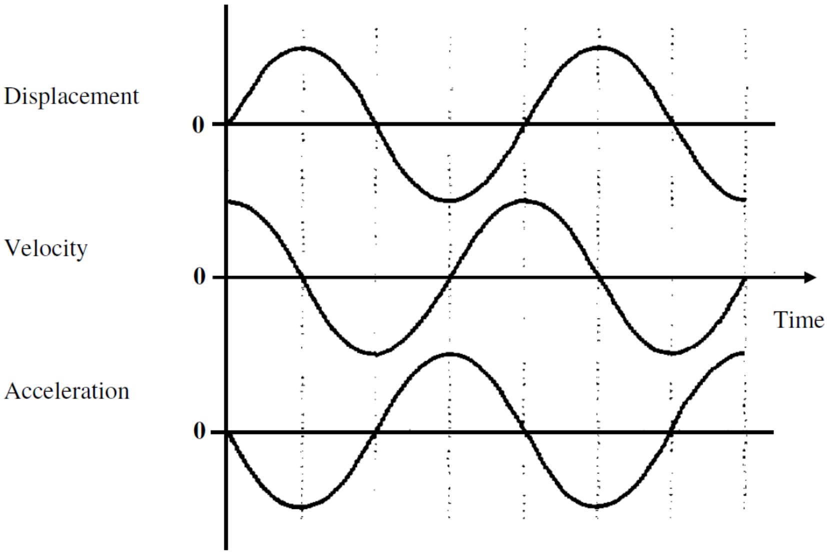

Figure 3: For a body executing SHM, these graphs are true.

- SHM is oscillating motion where the acceleration is:
  - proportional to displacement
  - in the opposite direction to displacement
- Thus the graphs of displacement and acceleration are in antiphase.
- The definition of SHM leads to $a = -\omega^2 x$ (a=amplitude, x=displacement)
  - General solution: $x = A\sin(\omega t + \phi)$ where ϕ is phase difference between t=0 and x=0
  - If timing starts at the centre ie $x = 0$, $x = A\sin(\omega t)$
  - whereas if timing starts at $x = +A$, $x = A\cos(\omega t)$ works.

$$v = \pm\omega\sqrt{A^2 - x^2}$$

8.1 Mass—spring system

$$T = 2\pi\sqrt{\frac{m}{k}}$$

T is increased by adding mass or using a weaker spring. Note that it does not depend on g.

8.2 Simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T is increased by increasing the length of the pendulum. Note the 'small angle approximation' — the angle of swing must be less than 10°.

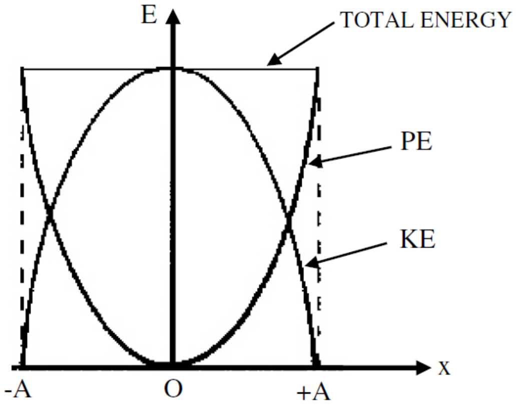

Figure 4: Variation of energy with displacement

8.3 Variation of energy with displacement

$$E_p = \frac{1}{2}kx^2$$

$$E_k = \frac{1}{2}k(A^2 - x^2)$$

$$E_{total} = \frac{1}{2}kA^2$$

8.4 Damping

- Light damping — T independent of amplitude so T remains constant as amplitude decreases. Amplitude gradually decreases by the same fraction each cycle.
- Critical damping — the system returns to equilibrium in the shortest possible time without overshooting
- Heavy damping — so strong that the displaced object returns to equilibrium much more slowly than if the system is critically damped — no oscillation occurs.

8.5 Forced vibrations and resonance

When a system oscillates without a periodic (driving) force applied, it oscillates at its natural frequency. Forced vibrations occur when a periodic force is applied to a system.

- As the applied frequency increases from 0:
  - amplitude of oscillation increases until a maximum is reached at a particular frequency — this is the resonant frequency — and the amplitude then decreases again.
  - phase difference between the displacement and the periodic force increases from 0 to $\frac{\pi}{2}$ at the maximum amplitude, and then from $\frac{\pi}{2}$ to π as frequency increases further.

When the system oscillates with maximum amplitude the phase difference between the displacement and the periodic force is $\frac{\pi}{2}$. The periodic force is then exactly in phase with the velocity of the system and resonance occurs.

- The lighter the damping:
  - the greater the maximum amplitude at resonance
  - the closer the resonant frequency to the natural frequency
  - hence the peak on a resonance curve will be much sharper with lighter damping.
- As the applied frequency becomes much larger than the resonant frequency:
  - amplitude of oscillations decreases more and more
  - phase difference between displacement and periodic force increases from $\frac{\pi}{2}$ until the displacement is π out of phase with the force.
- For an oscillating system with little to no damping, at resonance the applied frequency of the periodic force = the natural frequency of the system.

9 Gravitational fields

A force field is a region in which a body experiences a non-contact force. A force field can be represented as a vector, the direction of which must be determined by inspection.

- Gravity is a universal attractive force which acts between all matter.

  $$\text{magnitude of a force between point masses, } F = \frac{Gm_1m_2}{r^2} \text{ where } G \text{ is the gravitational constant}$$

- A gravitational field can be represented by field lines — also known as lines of force. This is the path followed by a small mass placed close to a massive body.
  - Note that for a radial field, the field lines point towards the centre. In a uniform field eg close to the Earth's surface, field lines act straight down — parallel to each other and evenly spaced.
- The gravitational field strength, g, is the force per unit mass on a small test mass placed in the field. $g = \frac{F}{m}$
- In a radial field, the magnitude of $g = \frac{GM}{r^2}$

9.1 Gravitational potential

- Gravitational potential at a point is the gravitational potential energy per unit mass of a small test mass.
  - This is equal to the *work done per unit mass to move an object from infinity (where potential = 0) to that point.*

    $$\text{gravitational potential, } V = \frac{W}{m} \text{ unit: J kg}^{-1}$$

    $$\text{work done moving mass } m\text{: } \Delta W = m\Delta V$$

    $$\text{gravitational potential in a radial field: } V = -\frac{GM}{r}$$

  - The negative sign is due to the reference point being infinity, and the fact that other than at infinity the force is in fact attractive.
- ΔV can be found from the area of a g—r graph
- Equipotentials are surfaces of constant potential — no work needs to be done to move along an equipotential surface.
- Potential gradient at a point in a gravitational field is the change of potential per metre at that point
- In general, for ΔV over a small distance Δr, potential gradient $= \frac{\Delta V}{\Delta r}$
- Gravitational field strength is the negative of potential gradient:

  $$g = -\frac{\Delta V}{\Delta r}$$

  the minus sign shows g acts in the opposite direction to potential gradient.
- g—r graph for a planet of radius R shows up to R, g increases at a constant rate. However, outside R it follows an inverse-square law — at $2R$, g is $\frac{1}{4}$ of the value at the surface.
- Gravitational potential is similar, but starts only at R. The graph goes from a negative value (potential at surface) and tends towards zero, at a non-linear rate.

9.2 Orbits and satellites

If an object is moving parallel to a planet's surface at the correct speed such that the centripetal force required is matched exactly by the force of gravity, it will orbit.

For a satellite orbiting at distance r from the centre of a planet: $\frac{GM_{planet}m}{r^2} = \frac{mv^2}{r} = mr\omega^2$ showing m irrelevant

- For geostationary orbit, $T_{sat} = T_{planet}$, so for earth $T \approx 86\ 400$ s.
  - $T = \frac{2\pi}{\omega}$

9.2.1 Kepler's 3$^{\text{rd}}$ Law proof & derivation

For an object in orbit around mass M:

1. $\frac{GM}{r^2} = r\omega^2$ so $\frac{GM}{r^3} = \omega^2$
2. Combining with $T = \frac{2\pi}{\omega}$ gives $\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$, or $T^2 = \frac{4\pi^2}{GM}r^3$
3. Everything is constant except T and r, meaning $T^2 \propto r^3$ — Kepler's 3$^{\text{rd}}$ Law
4. To further prove K3L, if $T^2 = \frac{4\pi^2}{GM}r^3$, taking logarithms gives $\log(T^2) = \log(\frac{4\pi^2}{GM}r^3)$
5. $\log(T^2) = \log(\frac{4\pi^2}{GM}) + \log(r^3)$
6. $2\log(T^2) = 3\log(r) + \log(\frac{4\pi^2}{GM})$
7. $\log(T^2) = 1.5\log(r) + 0.5\log(\frac{4\pi^2}{GM})$
8. so $\log(T^2) = 1.5\log(r) + \log(\sqrt{\frac{4\pi^2}{GM}})$
9. Hence a graph of $\log T$ against $\log r$ has gradient 1.5 and positive y-intercept of $\frac{2\pi}{\sqrt{GM}}$

9.2.2 Escape velocity

For an object to go into orbit once launched rather than fall back to Earth, it must never run out of kinetic energy. So supplied $\frac{E_k}{m} \geq V$.
Equating E_k and $V \cdot m$ allows us to work out that

$$\text{escape velocity, } v = \sqrt{\frac{2GM}{r}}$$

9.2.3 Energy considerations

A satellite has $E_k = \frac{1}{2}mv^2$. Equating forces in orbit gives $\frac{mv^2}{r} = \frac{GMm}{r^2}$ or $v^2 = \frac{GM}{r}$.

$$\text{Hence to be in orbit, } E_k = \frac{GMm}{2r}$$

$$\text{Potential energy is calculated from gravitational potential: } E_p = -\frac{GM}{r} \cdot m$$

$$\text{The } \underline{\text{total energy}} \text{ is the sum: } E_T = \frac{GMm}{2r} + (-\frac{GMm}{r})$$

$$E_T = -\frac{GMm}{2r}$$

10 Electrostatics

$$\text{Force between point charges in vacuo, } F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$$

- ε_0 = permittivity of free space
  - air can be treated as a vacuum when calculating force between charges
  - for a charged sphere, charge may be considered to be concentrated at the centre
- Electric fields can be represented by field lines — the direction of which is positive to less positive.
  - An electric line of force is the path along which a free positive charge would tend to move.
- Electric field strength at a point in an electric field is the force exerted by the field by a unit positive charge placed at that point

  $$\text{electric field strength, } E = \frac{F}{Q} \quad \text{unit: N C}^{-1} \text{ or V m}^{-1}$$

  Therefore the force exerted on charge Q at a point is given by $F = EQ$

  $$\text{magnitude of field strength in a \underline{uniform} field, } E = \frac{V}{d}$$

  - This can be derived from the work done moving a charge between the plates: $Fd = Q\Delta V$

  $$\text{field strength in a \underline{radial} field, } E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

10.1 Electric potential

The electric potential at a certain position in any electric field is the *'work done per unit positive charge on a positive test charge when it is moved from infinity to that position'*. Hence electric potential = 0 at infinity.

$$\text{Electric potential, } V = \frac{\text{work done, } W}{Q} \quad \text{unit: J C}^{-1}$$

$$\text{Work done moving charge } Q,\ \Delta W = Q\Delta V$$

$$\text{magnitude of electric potential in a \underline{radial} field, } V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Unlike gravitational potential, electric potential is a scalar quantity.

The electric potential of a positively charged particle increases as it moves to a point at higher potential — it gains energy from work having to be done to move it against electrostatic repulsion.

Potential difference between two points in an electric field is equal tot eh work done in moving a unit positive charge from the point at lower potential to the point at higher potential.

- The potential gradient at any position in an electric field is the change in potential per unit change of distance in a given direction.

  $$\text{electric field strength, } E = -\text{potential gradient} = -\frac{\Delta V}{\Delta r}$$

10.1.1 Graphical representations of E and V with r

- E—r graph follows an inverse-square law as $E \propto \frac{1}{r^2}$, but there is no electric field strength inside the charged sphere itself.
  - Hence graph starts at r rather than 0, and rapidly approaches 0.
  - ΔV can be found from the area under this graph as $E = -\frac{\Delta V}{\Delta r}$
- V—r graph is constant from 0 to r, then falls at a rate lesser than E—r graph as $V \propto \frac{1}{r}$

10.1.2 Projectile movement

A charged particle aimed through a uniform field will accelerate in one plane only, resulting in a parabolic arc similar to a ball thrown horizontally on Earth.

Relative strength: electric forces in a hydrogen atom are approximately 10^{39} times stronger than the gravitational forces acting.

11 Capacitance

A capacitor is any device used to store charge. The capacitance of an isolated conductor is the ratio of charge stored to the change in electric potential.

$$\text{capacitance, } C = \frac{Q}{V} \text{ unit: Farad, F}$$

$$\text{For a } \underline{\text{parallel plate}} \text{ capacitor, } C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

11.1 Energy stored

Charging a capacitor means transferring charge from the plate at lower potential to the plate at higher potential, which requires energy. Thus work done in charging = energy stored.

If a capacitor is charged to V by Q then the area under a V—Q graph gives the work done.

$$\text{work done, } W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

11.2 Discharging

$$\text{Charge left on a capacitor } t \text{ s after it starts discharging, } Q = Q_0 e^{-\frac{t}{RC}}$$

For a discharging capacitor the graphs of charge, voltage and current against time all have the same shape, so this formula works for V and I too.

$$\text{The } \underline{\text{time constant}} \text{ is } t \text{ taken for } Q \text{ to fall to } \frac{1}{e} \text{ of its previous value. } T = RC$$

From this, we can calculate that the time for charge or voltage to half in value is $0.693RC$.

11.3 Charging

The rate of charge leaving from or arriving on a capacitor depends on how much charge is already there. More work needs to be done to push electrons onto a partially charged capacitor than an empty one.

$$\text{For a charging capacitor, } Q = Q_0(1 - e^{-\frac{t}{RC}})$$

The graphs of Q and V against t show that charge & voltage increase rapidly at first, but the rate of change decreases as a maximum is approached. This means this equation works for V as well as Q, but not I (which looks the same for both a charging and discharging capacitor). Note that these don't work if current is kept constant!

Increasing R leads to a shallower charging or discharging curve which takes longer to reach its maximum or minimum. R decreases the current — decreasing the rate of flow of charge.

$$I = \frac{Q}{t}$$

11.4 Polarised molecules

Some molecules have one part more positive and another more negative — they are polarised.

If a polarised molecule is placed in an electric field, the two ends respond differently to the field, moving in opposite directions, rotating the molecule until it lines up with the field.

12 Electromagnetism

12.1 Magnetic flux density

Force on a current-carrying wire when field is perpendicular to current, $F = BIl$

- Fleming's left hand rule can be used to calculate the direction in which the wire will move.
  - thumb — thrust
  - first finger — field
  - second finger — current

The strength of a magnetic field is given by its flux density, B, which is measured in Tesla, T.

Flux density is a vector — its direction is along a tangent to the field line at that point. Its magnitude is represented by the density of magnetic field lines.

One Tesla is defined as *'the flux density of the field that produces a force of 1 N on a unit length of conductor carrying a current of 1 A perpendicular to the field.'*

- Note that for a magnetic field, field lines always run North—South
- The right hand grip rule gives the direction of field lines where the thumb points in the direction of current flow
  - $\otimes$ represents current flowing into the page, and $\odot$ current flowing out of the page. Imagine a dart flying.

Magnetic flux passing through area A perpendicular to a field, $\phi = BA$ unit: Webers, Wb

If area is not perpendicular to the field then $\phi = BA\cos\theta$ where θ is the angle to the normal to the area, or $\phi = BA\sin\theta$ where θ is to the plane of the area. Consider the graphs and where you would expect a maximum to occur.

$$\text{magnetic flux linkage } = N\phi \text{ where } N \text{ is the number of turns cutting the flux}$$

12.2 Charges in a magnetic field

Force on charged particles moving in a magnetic field, $F = BQv$ when field perpendicular to v

Charged particles in a magnetic field follow a circular path. The direction of the force on a positive charge is given by Fleming's LHR, and the force = centripetal force required to maintain this motion.

$$\text{so for a charged particle in a magnetic field, } BQv = \frac{mv^2}{r} = mr\omega^2$$

The cyclotron is an application of this phenomenon (magnetic deflection). Two D-shaped electrodes are separated by a small gap in an evacuated chamber placed in the uniform magnetic field of a large electromagnet.

Charged particles produced by an ion source at the centre enter one D and move in a circular path due to the field. A high-frequency alternating current is connected between the Ds, with frequency such that its polarity reverses at the same rate as the particles cross from one D to another. The energy is of the particle is increases every time it crosses from one D to the other, the radius of orbit increases as energy increases and the beam finally emerges tangentially from the cyclotron.

At radius r, magnetic force = centripetal force, so $r \propto v$

12.3 Electromagnetic induction

When a wire cuts through magnetic field lines, an electromotive force is induced in the wire.

12.3.1 Lenz's Law

'The direction of the induced current is such that it opposes the motion producing it.'

12.3.2 Faraday's Law

'The magnitude of the induced emf is proportional to the rate of change of flux linked with that circuit, or the rate at which magnetic flux is cut'

$$\text{Combined with Lenz's Law: } \varepsilon_{ind} = -\frac{\Delta}{\Delta t}(\text{flux linkage})$$

12.3.3 Flux linkage

$$\text{As flux linkage} = N\phi,\ \varepsilon_{ind} = \frac{\Delta}{\Delta t}(N\phi)$$

Given $\phi = BA$, the flux linking a coil $= BAN$ and:

$$\varepsilon_{ind} = \frac{\Delta}{\Delta t}(BAN)$$

$$\text{or average } \varepsilon_{ind} = \frac{\text{change in } BAN}{\text{time taken}}$$

Both of these assume the plane of the coil is perpendicular to the coil. Otherwise flux linking coil $= BAN\cos\theta$ or $BAN\sin\theta$ (consider maxima)

12.3.4 emf induced in a moving conductor

$$\varepsilon_{ind} = Blv$$

Fleming's right hand rule gives the direction of the induced current if a complete circuit. If asked to label emf consider the conductor just as any other source — in a wire connected between the terminals, current would flow from positive to negative.

12.3.5 emf induced in a rotating coil

For θ between the normal to the coil and the field, flux linking the coil is given by $\phi = BAN\cos\theta$. For a constant rate of rotation $\theta = \omega t$ where ω is the angular speed in rad s^{-1}.

Therefore $\phi = BAN\cos(\omega t)$.

$$\text{Combining with Faraday's Law, } \varepsilon_{ind} = BAN\omega\sin(\omega t)$$

The induced emf is therefore a sine wave with peak value $BAN\omega$. The faster the coil is rotated, the greater the peak. This very much depends on when timing starts however, so consider maxima. Note that maximum ε_{ind} occurs where $\frac{d\phi}{d\theta}$ is greatest ie wires are cutting the most field lines.

13 Thermal physics

- Internal energy is that of an object's molecules due to their individual movements and positions
- It is increased through:
  - energy transfer by heating
  - work done on the object eg by electricity
- If the internal energy is constant:
  - there is no energy transfer by heating and no work is done
  - or energy transfer by heating and work done balance each other out

- The first law of thermodynamics:in general, when work is done on or by an object and/or energy is transferred by heating:
  - the change of internal energy of the object = the total energy transfer due to work done and heating
  - the directions of the energy transfers are important in determining whether there will be a decrease or increase in internal energy
- The internal energy of the object is the sum of the random distribution of the kinetic and potential energies of its molecules.
- Celsius
  - 0 °C — ice point, temperature of pure melting ice
  - 100 °C — steam point, temperature of steam at standard pressure (100 kPa)
- Absolute scale
  - 0 K — absolute zero (−273 °C)
  - 273 K — triple point of water, where ice, water and water vapour coexist in thermodynamic equilibrium (ie objects at same temperature so no energy transfer by heating)
- Absolute zero is the minimum temperature any object could have — an object at 0 K has minimum internal energy regardless of its composition.
- A graph of gas pressure against temperature will always pass through the x-axis at absolute zero.

13.1 Specific heat capacity

Specific heat capacity, c, is the energy needed to raise the temperature of a unit mass of a specific substance by 1 K without a change of state. c has the unit J kg^{-1} K^{-1}.

$$\text{energy, } Q = mc\Delta T$$

For continuous flow heating one must consider the energy supplied per second.

13.2 Change of state

A solid becomes a liquid thanks to energy being supplied at its melting point, where its atoms vibrate so much they break free from each other. The energy needed to melt a solid already at melting point is the latent heat of fusion.

Latent heat is released when a liquid solidifies because the liquid's molecules slow down as it cools until the temperature reaches the melting point. At this point the molecules move slowly enough for them to lock together — some of the latent heat released keeps the temperature at the melting point until all the liquid has solidified.

When a liquid becomes a gas the molecules gain enough energy to overcome the bonds holding them close together. The energy needed to vaporise a liquid is the latent heat of vaporisation.

Latent heat is then released when a vapour condenses as its molecules slow down.

Some solids vaporise directly when heated: sublimation.

In general much more energy is needed to vaporise a substance than to melt it.

$$Q = ml, \text{ where } l \text{ is the specific latent heat and is measured in J kg}^{-1}$$

During a change of state the potential energies of the particle ensemble are changing but not the kinetic energies.

13.3 Ideal gases

The pressure of a gas is the force per unit area that the gas exerts normally on a surface. It is dependent on temperature, the volume of the gas container, and the mass of gas in the container.

- Boyle's Law: pV = constant for fixed m and constant T
  - Pressure of gas at constant T increased by reducing its volume as gas molecules travel less distance between impacts — hence more impacts per second and so greater pressure
- Charles' Law: $V \propto T \implies \frac{V}{T}$ = constant for fixed m and constant p
- Any change at constant pressure is isobaric — when work is done to change the volume of a gas, energy must be transferred by heating to keep pressure constant and so:

$$\text{work done, } W = p\Delta V$$

- Pressure law: $p \propto T \implies \frac{p}{T}$ = constant for fixed m and constant V
  - pressure of a gas at constant volume increased by raising its temperature — raises average speed of molecules and so impacts on the container walls are harder and more frequent: raising pressure.

    Note: T must always be in Kelvin.

13.4 Ideal gas law

- A number of assumptions must be made, including:
  1. Intermolecular forces are negligible except during a collision
  2. Volume of the molecules negligible compared to volume of gas
  3. Collisions between molecules and between molecules and the container walls are perfectly elastic
  4. Duration of a collision negligible compared to time between collisions.
  5. Laws of Newtonian Mechanics apply
  6. All molecules of a particular gas are identical
  7. The motion of molecules is random
  8. There is a large number of molecules
- Brownian motion can be seen when smoke particles are observed with a microscope — they move unpredictably. The motion of each particle is because it is bombarded unevenly and randomly by individual molecules — thus particles experience forces which change in magnitude and direction at random.
  - showed the existence of molecules and atoms

13.4.1 Moles

- Avogadro's constant, N_A, is the number of atoms in 12 g of Carbon-12.
- One atomic unit (au) is $\frac{1}{12}$ the mass of a Carbon-12 atom.
- 1 mol of a substance of identical particles is the quantity of the substance that contains N_A particles.
- Molar mass of a substance is the mass of 1 mol of the substance

$$\text{number of moles} = \frac{\text{mass of substance}}{\text{molar mass}}$$

$$\text{number of molecules} = N_A \times \text{number of moles}$$

- An ideal gas is one which obeys Boyle's Law.

$$\text{combining gas laws: } \frac{pV}{T} = \text{ constant for fixed } m \text{ of ideal gas}$$

$$\text{For 1 mol of any ideal gas, } \frac{pv}{T} = R$$

- Graph of pV against T for n mol is a straight line through absolute zero and has gradient nR

$$\text{hence } pV = nRT \text{ where } n \text{ is number of moles}$$

$$pV = NkT \text{ where } N \text{ is the number of molecules}$$

13.5 Derivation of kinetic theory equation

1. Initial momentum of a gas particle is mv
   Momentum after contact with the container wall is $-mv$

$$\Delta\text{momentum} = 2mv$$

2. Time between collisions: $v = \frac{s}{t} = \frac{2l_1}{t}$

$$\text{time between collisions, } t = \frac{2l_1}{v}$$

3. Newton's second law:

$$F = \frac{\Delta mv}{\Delta t} = 2mv \times \frac{v}{2l_1} = \frac{mv^2}{l_1}$$

4. Pressure:

$$p = \frac{F}{A} = \frac{mv^2}{l_1} \times \frac{1}{l_2 l_3} = \frac{mv^2}{l_1 l_2 l_3} = \frac{mv^2}{V}$$

5. However, there are <u>many</u> gas particles:

$$p = \frac{mv_1^2}{V} + \frac{mv_2^2}{V} + \frac{mv_3^2}{V} \cdots + \frac{mv_n^2}{V}$$

$$p = \frac{m}{V}(v_1^2 + v_2^2 + v_3^2 \ldots + v_n^2)$$

   Need to use an average:

$$\text{average } v,\ (c_{RMS})^2 = \frac{v_1^2 + v_2^2 + v_3^2 \ldots + v_n^2}{n}$$

$$n(c_{RMS})^2 = v_1^2 + v_2^2 + v_3^2 \ldots + v_n^2$$

$$\therefore \text{overall: } p = \frac{mn(c_{RMS})^2}{V}$$

6. However, particles do not just move in one plane $\therefore$ we are dealing with a component in one of three planes — a third of the total magnitude.

$$p = \frac{mn(c_{RMS})^2}{3V}$$

7. $mn =$ total mass, M:

$$p = \frac{M(c_{RMS})^2}{3V}$$

$$\rho = \frac{M}{V}$$

$$\therefore p = \frac{1}{3}\rho(c_{RMS})^2$$

13.6 Molecules and kinetic energy

For an ideal gas its internal energy is due only to the kinetic energy of the molecules of the gas

$$\text{kinetic energy of a molecule} = \frac{\text{total } E_k \text{ of all the molecules}}{\text{total number of molecules}} = \frac{\frac{1}{2}m(c_1^2 + c_2^2 \ldots c_N^2)}{N} = \frac{1}{2}m(c_{RMS})^2$$

The higher the temperature of a gas the greater the mean kinetic energy of a molecule of the gas.

$$\text{for an ideal gas, mean } E_k \text{ of a molecule} = \frac{3}{2}kT$$

$$\text{total } E_k \text{ of } n \text{ mol of ideal gas} = \frac{3}{2}nRT = (\text{internal energy})$$

14 Nuclear physics

- Rutherford used α radiation, fast-moving positively-charged particles, to probe the atom. A beam of α was directed at a thin sheet of metal foil.
  - Expectation: beam might be scattered slightly if positive charge was spread throughout each atom
  - Reality: some particles bounced back
    - some α passed straight through, with about 1 in 2000 being deflected
    - around 1 in 10 000 were deflected through angles of more than 90°
- This could be explained by assuming every atom had a 'hard centre' much smaller than the atom itself.
  - most of the atom's mass is concentrated in a small region, the nucleus
  - the nucleus must be positively charged as it repels α particles which approach it too closely
- Rutherford also showed $|Q|$ on a nucleus $= Ze$ where e is the charge on an electron and Z is the element's proton number.
- The diameter of a nucleus is of the order 10^{-15} m

14.1 α, β and γ radiation

- Becquerel found that uranium salts emit radiation which can penetrate paper and darken photographic film.
- Rutherford found radiation:
  - ionised air — making it conduct electricity
    - allowing a detector to be developed which measures radiation from its ionising effect
  - was of two types:
    - α: easily absorbed
    - β: more penetrating
    - (γ, which is even more penetrating, was discovered later)
- A magnetic field deflects α and β in opposite directions — so could conclude that α is positively charged and β negative. γ is not deflected but consists of high energy photons.

14.1.1 Experiments

- Ionisation — the ionising effect of each type can be investigated with an ionisation chamber (containing air at atmospheric pressure) and a picoammeter
  - ions created in the chamber are attracted to the oppositely charged electrode where they are discharged — as a result of the ionisation electrons pass through the picoammeter
  - $I_{pA} \propto$ number of ions per second
  - α most ionising, γ least as no charge
- Cloud chamber — due to the ionisation of saturated air α or β particles leave a track of minute droplets
  - α: straight tracks radiating from the source which are easily visible. Those from the same source are all of the same length.
    - α from the same source have the same range in air as each other as the are always emitted with the same E_k — each α and the nucleus that emits it move apart with equal and opposite momentum.
  - β: wispy tracks that are easily deflected as a result of collision with air molecules. Harder to see as they are less ionising.
    - β particles have varying ranges in air because a V_e or $\overline{V_e}$ is emitted as well — the nucleus, β and V_e or $\overline{V_e}$ share the energy released in variable proportions.
- Geiger—Muller tube
  - sealed metal tube containing argon gas at low pressure. Mica window allows α and β to enter, and γ can also enter through the tube wall.

  - metal rod down the centre is positive and the tube wall negative
  - ionising radiation enters, ionising gas along its track. Negative ions go to the rod, and positive to the wall. Ions accelerate and collide with other ions, resulting in more ions
  - in a short time many ions are created and discharged at the electrodes, giving a pulse of charge which results in a voltage pulse which is recorded.

- Absorption
  - α: absorbed completely by paper or thin metal foil; range of a few centimetres in air
  - β: around 5 mm of metal; range approx. 1 m in air
  - γ: several centimetres of lead needed; unlimited range in air but follows the inverse square law

14.1.2 Equations for radioactive change

$$
\begin{gathered}
{}^{A}_{Z}X \rightarrow {}^{4}_{2}\,\alpha + {}^{A-4}_{Z-2}\,Y \\
{}^{A}_{Z}X \rightarrow {}^{0}_{-1}\,\beta + {}^{A}_{Z+1}\,Y + \overline{V_e} \\
{}^{A}_{Z}X \rightarrow {}^{0}_{+1}\,\beta + {}^{A}_{Z-1}\,Y + V_e
\end{gathered}
$$

In electron capture a proton-rich nucleus can captures an inner-shell electron, causing a proton in the nucleus to change into a neutron with the emission of an electron neutrino at the same time. The inner shell vacancy is then filled by an outer shell electron, resulting in emission of an X-ray photon.

$${}^{A}_{Z}X + {}^{0}_{-1}\,e \rightarrow {}^{A}_{Z-1}\,Y + V_e$$

In γ emission no change occurs in the number of protons or neutrons of a nucleus. A γ photon is emitted if a nucleus has excess energy after it has emitted an α or β^- particle.

14.2 Radiation safety

Ionising radiation is dangerous as it can damage living cells — this radiation includes X-rays, protons and neutrons as well as α, β and γ.

- can destroy cell membranes causing cell death
- damage vital molecules such as DNA directly, or indirectly by creating 'free radical' ions that react with vital molecules
  - normal cell division affected & nuclei damaged
  - may cause cells to divide & grow uncontrollably → tumour
  - damaged DNA in sex cells → mutation which may be passed down
- there no evidence of a threshold below which ionising radiation is safe

14.2.1 Monitoring exposure

- Film badge — a strip of photographic film in a lightproof wrapper. Different areas are covered by different absorbers.
  - when film developed the exposure can be estimated from the darkness of each area — the different absorbers allowing the various types of radiation to be distinguished.
- Dose is measured in terms of energy absorbed per unit mass of the matter from the radiation.
- Dose equivalent is measured in Sieverts, Sv, and is the dose due to 250 kV X-rays that would have the same effect.
- Background radiation occurs naturally due to cosmic radiation and from radioactive materials in rocks, soil and the air. It varies with location due to local geological filters. Other sources of background radiation are air travel, nuclear power, weapons tests, and medical use of radioactivity (the largest man-made BG source)

- ALARP is 'as low as reasonably practical' — risks are always reduced by increasing distance from the source and decreasing the time of exposure.

- Storage of radioactive materials
  - lead-lined containers — most sources produce γ as well as α or β so lining must be thick enough to reduce γ from the source(s) inside to background
  - containers themselves must be under 'lock and key' and records kept
- Using radioactive materials
  - do not allow contact with skin
  - solid sources are transferred using tools such as tongs, glove-box or using robots — ensures the material is as far from the user as possible
  - liquid, gas or powder sources are always in sealed containers
  - sources must be used no longer than necessary

14.3 Radioactive decay

- Half life, $T_{\frac{1}{2}}$, of a radioactive isotope is the time taken for the mass of the isotope to decrease to half the initial mass. It is the same as the time taken for the number of nuclei of that isotope to halve.
- The mass of X increases exponentially because radioactive decay is exponential & number of nuclei that decay $\propto$ number of nuclei of X remaining.
- Activity, A (also $\frac{\Delta N}{\Delta t}$), of a radioactive isotope is the number of nuclei of the isotope that disintegrate per second — ie the rate of change of the number of nuclei of the isotope.
  - unit is the Becquerel, Bq — 1 Bq = 1 disintegration per second
- For a source of activity A emitting particles or photons of the same energy E the energy per second released by radioactive decay (ie the power) $= AE$
  - If a source emitting only α is in a sealed container which absorbs all the radiation, then the container gains thermal energy equal to the energy transferred from the source

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$$A = \lambda N$$

$$N = N_0 e^{-\lambda t} \text{ (note applies for } A,\ m,\ C \text{ (corrected count rate) and } \frac{\Delta N}{\Delta t} \text{ too)}$$

The decay constant, λ, is the probability of an individual nucleus decaying per second

$$\text{probability per unit time} = \frac{\frac{\Delta N}{N}}{\Delta t} = \lambda, \text{ unit: s}^{-1}$$

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

14.4 Radioactive dating

- Carbon
  - living trees and plants contain a small percentage of carbon-14 which is formed by cosmic rays knocking neutrons out of nuclei which then collide with nitrogen nuclei
  - $T_{\frac{1}{2}}$ of 5570 years so negligible decay in the life of the organism
  - once dead no more carbon-14 taken in so proportion of carbon-14 decreases as it decays
  - activity, $A \propto N$ so knowing A allows age of a sample to be calculated provided A of the same mass of living wood known

- Argon
  - ancient rocks contain trapped argon gas as a result of decay of potassium ${}^{40}_{19}\mathrm{K}$ into argon ${}^{40}_{18}\mathrm{Ar}$ through electron capture
$$ {}^{40}_{19}\mathrm{K} + {}^{0}_{-1}e \rightarrow {}^{40}_{18}\mathrm{Ar} + V_e $$
  - the potassium also decays through β^- decay to form calcium ${}^{40}_{20}\mathrm{Ca}$ – a process eight times more likely
$$ {}^{40}_{19}\mathrm{K} \rightarrow {}^{0}_{-1}\beta + {}^{40}_{20}\mathrm{Ca} + \overline{V_e} $$
  - effective $T_{\frac{1}{2}}$ of ${}^{40}_{19}\mathrm{K}$ is 1250 million years — age of rock can be calculated from proportion of argon-40 to potassium-40
  - for every N potassium atoms present, if there is 1 argon atom there must have originally been $N+9$ potassium atoms
    - 1 decayed to ${}^{40}_{18}\mathrm{Ar}$ and 8 into ${}^{40}_{20}\mathrm{Ca}$ + N remaining
    - can then use $N = N_0 e^{-\lambda t}$

14.5 Industrial uses of radioactivity

- Radioactive tracers are used to follow the path of a substance through a system. The isotopes used should:
  - have $T_{\frac{1}{2}}$ stable enough for the necessary measurements to be made and short enough to decay quickly after use
  - emit β or γ so it can be detected outside the flow path
- Engine wear
  - rate of wear of a piston ring can be measured by fitting a ring that is radioactive: as the ring slides, radioactive atoms transfer from the ring to the engine oil
  - measure A of oil, m of radioactive material transferred from the ring can be determined and so rate of wear calculated
  - a metal ring can be made radioactive by exposing to neutron radiation in a nuclear reactor — each nucleus that absorbs a neutron becomes unstable and disintegrates through β^-
- Thickness of a manufactured sheet
  - detector measures amount of radiation passing through the foil — if too thick then reading drops; feedback from detector to control system increases roller pressure. Same vice-versa.
  - β source with long $T_{\frac{1}{2}}$ used
- Powering remote devices
  - satellites, weather stations etc can be powered using a radioactive isotope in a thermally insulated sealed container that absorbs all the radiation emitted
  - thermocouple attached to container produces electricity as a result of the container heating up
  - $A = \lambda N$, each disintegration released E J so power = $\lambda N E$
  - source needs reasonably long $T_{\frac{1}{2}}$ so not replaced frequently, but need to balance against mass required for that power.

14.6 N—Z graph

Neutron number N against proton number Z for all isotopes. Graph shows that stable nuclei lie along a belt curving up with an increasing N:Z ratio from 0 to $N = 120$, $Z = 80$ approx.

- for light isotopes ($0 < Z < 20$) stable nuclei follow the line $N = Z$ — they have equal neutrons to protons.
- as Z increases beyond 20, stable nuclei have more neutrons than protons so N:Z increases. Extra neutrons bind nucleons together without introducing repulsive electrostatic forces.
- α emitters occur $Z > 60$, most with more than 80 protons and 120 neutrons. N:Z> 1 but nuclei too large to be stable as the strong force is less than the electrostatic force of repulsion between the protons.
- β^- emitters left of the stability belt with high N:Z

- β^+ emitters right of stability belt with low N:Z
- α emission moves the parent nucleus diagonally down and left on graph
- β^- — down and right
- β^+ — up and left

Many radioactive isotopes decay to form another which might itself be unstable. Unstable daughter nucleus may then decay again. Thus an unstable nucleus before becoming stable may undergo a series of isotopic changes where each change involves an emission of α or β. Naturally occurring radioactive isotopes decay through a series of such changes with one or more of the changes having a very long $T_{\frac{1}{2}}$

This can be represented by decay arrows on an N—Z graph — but note that it will not move away from the stable belt.

β^- emitters are manufactured by bombarding stable isotopes with neutrons, and β^+ by bombarding with protons of sufficient E_k to overcome electrostatic repulsion from nucleus.

14.7 Nuclear energy levels

- Following α, β emission or electron capture, an unstable nucleus may emit a γ photon. γ photon emission does not change the number of protons or neutrons in the nucleus but does allow it to lose energy
- this happens if the daughter nucleus is formed in an excited state after the previous decay state. Excited state is usually short-lived and the nucleus moves to its ground state either directly or via one or more lower-energy excited states.
- On a nuclear energy level diagram diagonal lines represent decays and vertical lines are dropping energy levels due to γ photon emission.

14.7.1 Technetium generator

- Used in hospitals to produce a source which emits only γ
- ${}^{99}_{42}\mathrm{Mo}$ decays via β^- to ${}^{99}_{43}\mathrm{Tc}$ which stays in its excited state long enough to be separated from the parent isotope
    - such a long-lived excited state is metastable
- ${}^{99}_{43}\mathrm{Tc}$ in the ground state is β^- emitter with $T_{\frac{1}{2}}$ 500 000 years & forms a stable product. ${}^{99}_{43}\mathrm{Tc}^{\mathrm{m}}$ (metastable) with no Mo present emits only γ photons
- The technetium generator consists of an ion exchange column containing ammonium molybdenate exposed to neutron radiation several days earlier to make a significant number of the Mo nuclei unstable. Sodium chloride solution is passed through the column and the solution that emerges contains ${}^{99}_{43}\mathrm{Tc}^{\mathrm{m}}$ nuclei
    - diagnostic uses include monitoring blood flow through the brain and use with the γ camera

14.8 Nuclear radius

14.8.1 High energy electron diffraction

- can be used to measure the diameter of different nuclides
- when a beam of high E electrons is directed at a thin solid sample of an element the incident electrons are diffracted by the nuclei in the foil
- beam generated by accelerating through a 100 MV+ p.d.
- electrons diffracted as their de Broglie λ is of the order 10^{-15} m — about the same as the diameter of a nucleus. A detector is used to measure the number of electrons per second refracted through different angles
- As θ of detector to zero-order beam increases, intensity decreases then increases slightly before increasing decreasing again

- scattering of the electrons by the nuclei occurs due to their charge — same as α deflection but attraction rather than repulsion — causes intensity to decrease as θ increases
- diffraction of the electrons by nuclei causes intensity maxima and minima to be superimposed on the scattering effect. Happens provided the de Broglie $\lambda \leq$ dimensions of nucleus. These superimposed intensity variations are on a much smaller scale.
- angle of the first minimum from the centre, $\theta_{\min}$, is measured and used to calculate diameter of the nucleus provided that the wavelength of the incident electrons is known.

14.8.2 Dependence of radius on mass number

$$R = R_0 A^{\frac{1}{3}}$$

- graph $\ln R$ against $\ln A$ — straight line with gradient $\frac{1}{3}$ and y-intercept $\ln R_0$
- R against $A^{\frac{1}{3}}$ gives straight line through origin, gradient R_0
- R^3 against A has gradient ${R_0}^3$

14.8.3 Nuclear density

$$\text{Assuming nucleus spherical } V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{\frac{1}{3}})^3 = \frac{4}{3}\pi {R_0}^3 A$$

- So nuclear volume, $V \propto$mass of nucleus $\therefore \rho_{\text{nucleus}}$ is constant independent of radius and is constant throughout the nucleus.
- Can conclude nucleons are separated by the same distance regardless of the size of the nucleus & therefore evenly separated
- Can calculate ρ_{nucleus} using the volume formula and knowledge that $m = Au$ where $u = 1$ atomic mass unit.

14.9 Nuclear energy

$$E = mc^2 \text{ applies to all energy changes of any object}$$

Such changes of energy tend to be insignificant other than in nuclear reactions — need to work to 6 or 7 s.f. when dealing with nuclear reactions.

$$\text{energy released, } Q = \Delta mc^2$$

In any change where energy is released the total mass after the change is always less than the total mass before — in the change some of the mass is converted to energy which is released.

- α decay: nucleus recoils when α particle released so energy released is shared between both. Applying conservation of momentum can show energy released is shared between the nucleus & α in inverse proportion to their mass.
- β decay: energy released shared in variable proportions between the β particle, the neutrino or antineutrino released and the nucleus. When β has max E_k the (anti)neutrino has negligible E_k in comparison. $E_{k\max}$ of β is slightly less than that released in decay because of the nucleus' recoil.
- Electron capture: nucleus emits neutrino which carries away the energy released in the decay, atom emits X-ray photon when the inner shell vacancy is filled.

14.9.1 Binding energy

- The binding energy of a nucleus is the work that must be done to separate a nucleus into its constituent neutrons and protons.
- When a nucleus forms from separate neutrons and protons energy is released as the strong force does work pulling the nucleons together. The energy released is equal to the binding energy of the nucleus.
  - because E released when nucleus forms from separate nucleons the mass of the nucleus is less than the mass of the separate nucleons.

    mass defect, $\Delta m =$ difference between mass of separated nucleons and mass of the nucleus
- for nucleus ${}^{A}_{Z}\text{X}$ of mass M, $\Delta m = Zm_p + (A - Z)m_n - M$

$$\text{binding energy of nucleus} = \Delta mc^2$$

14.9.2 Nuclear stability

The binding energy of each nucleon is different — binding energy per nucleon of a nucleus is the average work done per nucleon to remove all nucleons from a nucleus. Hence it is a measure of nuclear stability — the nucleus with more binding energy per nucleon is more stable.

- A graph of average binding energy per nucleon against nucleon number A shows a sharp increase up to 8.7 MeV/nucleon between $A = 50$ and $A = 60$, and then a slow, much shallower decrease.
  - nuclei in the range $A = 50$ to $A = 60$ are the most stable.
- energy is released in:
  - fission: large unstable nucleus splits into two fragments which are more stable than original. Binding energy per nucleon increases.
  - fusion: small nuclei fuse together to make a larger nucleus
    - fusion product has more binding energy per nucleon than the smaller nuclei so binding energy per nucleon also increases in fusion — provided $A \lessapprox 50$
- change in binding energy per nucleon is ≈ 0.5 MeV in fission and can be more than 10× as much in fusion.

14.9.3 Induced fission

Fission occurs when uranium-235 is bombarded with neutrons (induced fission). Plutonium-239 is the only other fissionable isotope, and is an artificial isotope formed by neutron bombardment of $^{235}_{92}\mathrm{U}$.

- Each fission event releases energy and 2-3 neutrons
- Fission neutrons are capable of causing further fission as a result of collision with a $^{235}_{92}\mathrm{U}$ nucleus
  - chain reaction is therefore possible. If each fission event releases 2 neutrons on average, after n generations of fission the number of fission neutrons would be $\approx 2^n$
- Energy is released when fission occurs as the fragments repel each other (each has a positive charge) with sufficient force to overcome the strong force which is trying to hold them together. Fragment nuclei & fission neutrons therefore gain E_k
  - fragment nuclei smaller & more tightly bound than parent — so more binding energy so more stable

$$\text{energy released} = \text{change in binding energy} = \Delta mc^2$$

Nuclear fusion can only occurs if the two nuclei that are to be combined collide at high speed — must overcome the electrostatic repulsion between the two nuclei so they can become close enough to interact through the strong force.

14.10 Thermal nuclear reactor

- Fuel rods contain enriched uranium consisting mostly of non-fissionable U-238 and 2-3% fissionable U-235. Natural uranium is 99% U-238
- Control rods absorb neutrons — depth of control rods in core is automatically adjusted to keep the number of fission neutrons constant such that exactly one fission neutron per fission event on average goes on to produce further fission (the critical condition)
  - keeps rate of release of fission energy constant
  - if control rods pushed in further they absorb more neutrons so number of fission events per second & rate of release of fission energy reduce.
- Moderator surrounds the fuel rods so as to slow down the fission neutrons (may take ≈ 50 collisions), otherwise they would be travelling too quickly to cause further fission.
  - 'thermal nuclear reactor' as fission neutrons slowed down to E_ks comparable to E_ks of the moderator atoms. In PWR the water acts as both moderator and coolant.
- For a chain reaction to occur the mass of fission material eg U-235 must be greater than critical mass
  - because some fission neutrons escape from the fuel without causing fission, and some are absorbed by other nuclei without fission. If mass of fissile material is less than the critical mass then too many fission neutrons escape because the surface area to mass ratio is too high.

14.10.1 Safety features

- Reactor core is thick steel — withstand high pressure & temperature. Absorbs β, some γ and neutrons from core
- Building with very thick concrete walls to absorb neutrons & γ
- Emergency shutdown — fully insert control rods to completely stop fission
- Sealed fuel rods inserted and removed using remote handling devices
  - note spent fuel rods are more radioactive than unused:
    - before: just U-238 & U-235 so only α (easily absorbed)
    - after: β and γ too due to many neutron-rich fission products
      also plutonium-239 which is an active α emitter that causes lung cancer if inhaled

14.10.2 Radioactive waste

- High-level radioactive waste such as spent fuel rods from a power station contain many different isotopes
  - must be removed by remote control and stored underwater in cooling ponds for up to a year as they continue to release heat
  - in UK, rods are then transferred in large steel cases to reprocessing plants where unused plutonium and uranium removed. Remaining material is radioactive waste & is stored in sealed containers in deep trenches
  - in some countries waste is ‘vitrified’ by mixing with molten glass & then stored as glass blocks in underground caverns
- Intermediate-level waste — materials with low activity & their containers are sealed in drums encased in concrete and stored in specially constructed buildings with reinforced concrete walls.
- Low-level waste such as lab equipment & protective clothing is sealed in metal drums and buried in large trenches.

15 Electronics

15.1 Discrete semiconductor devices

15.1.1 MOSFET

- Enhancement mode MOSFETs are voltage operated, have very large input resistance (> 50 MΩ) and large current gain.
- To obtain a drain current, V_{gs} is applied between the gate and source.
  - Output voltage does not change much until $V_{\mathrm{gs}} \gtrapprox 1$ V
  - drain current passes & MOSFET saturates ≈ 0.1 V when $V_{\mathrm{gs}} \approx 2$ V
  - any further increase in V_{in} (V_{gs}) has no effect on V_{ds} — MOSFET acts like a switch
- MOSFET selected needs to be able to operate at the supply voltage, pass sufficient drain current (I_{ds}), dissipate sufficient power & give a low R_{ds}.
- A MOSFET has a positive temperature coefficient — if its temperature increases, R_{ds} does too, and so I_{ds} decreases.
- Thanks to its very high input resistance (a result of the MOSFET's insulating metal oxide layer) a MOSFET could easily be used to buffer the output of a low power subsystem (draws very little current)
  - however, the metal oxide layer is very thin, and a MOSFET is susceptible to destruction by electrostatic charges accumulating on the oxide layer between the gate and source
- In some circuits. a high value resistor is placed from the gate of an n-channel MOSFET to 0 V: this helps prevent static charge from building up on the gate when there is no input, which could lead to damage or transient signals being passed.

15.1.2 Zener diode

- Cheap & convenient way of providing a stabilised power supply. They are used in reverse bias & the voltage across them is constant providing a minimum current (I_{min}) flows.
- At a certain breakdown voltage the reverse current increases suddenly. At this voltage the reverse current is limited by a series resistor so that the voltage across the Zener diode, V_{Z}, remains fairly constant over a wide range of reverse currents passing through the diode.
- Zener diodes are made with breakdown voltages from 2.7 V to 200 V
- As well as providing a stabilised power supply, Zener diodes can be used to:
  - prevent voltage difference in a system exceeding a chosen value
  - reduce a voltage by a certain amount irrespective of current flowing

$$I_{\mathrm{Z}\max} = \frac{P_{\max}}{V_{\mathrm{Z}}}$$

$$I_{\mathrm{load}} = I_{\mathrm{Z}} - I_{\mathrm{min}} \text{ (if } I_{\mathrm{load}} > I_{\mathrm{Z}} \text{ then } V_{\mathrm{Z}} \text{ will decrease)}$$

$$R_{\mathrm{min}} = \frac{V_{\max} - V_{\mathrm{Z}}}{I_{\mathrm{Z}}}$$

$$\text{normally, } P_R = I_{\mathrm{Z}}(V_{\max} - V_{\mathrm{Z}}) \text{ but under fault, } P_{R(\mathrm{fault})} = I_{\mathrm{Z}} \times V_{\max}$$

15.1.3 Photodiode

- A photodiode is a semiconductor diode which conducts only when light is incident on its P—N junction. When connected in reverse bias it is working in 'photoconductive' mode.
- The depletion region at the P—N junction has a potential difference across it when reverse biased. When photons incident on the depletion region electron—hole pairs are formed
- Electric field due to potential difference sweeps holes to the anode and pairs to the cathode such that a 'reverse photocurrent' flows. This is proportional to light intensity

- Photodiodes can be manufactures with spectral responses in different parts of the electromagnetic spectrum. A photodiode is most responsive at the peak of a graph of relative response against wavelength.

$$\text{photosensitivity} = \frac{\text{photocurrent generated}}{\text{power incident}}, \text{ unit: A W}^{-1}$$

Photodiodes in photoconductive mode have many applications eg in optical fibre communications, light meters, smoke detectors, photocopiers.

- A scintillator is a material that produces a flash of light when a particle such as an ion, electron, α or high-energy photon passes through.
- If coupled to a photodiode then the number of light pulses can be detected & amplified and the energy of the particle through the scintillator measured.
- A single particle event depositing energy in the scintillator may typically produce a few thousand photons; this is the 'light yield' and is the number of photons per MeV
- For maximum efficiency the spectral response of the photodiode needs to be matched as closely as possible to the wavelength produced by the scintillator

$$\text{scintillator efficiency} = \frac{\text{total } E \text{ of light photons produced in scintillator}}{E \text{ deposited in scintillator by incident particles/photons}} (\times 100)$$

15.1.4 Hall effect sensor

- The Hall effect arises from a magnetic force on and hence movement of charge carriers across a current-carrying semiconductor material, so creating an electric field across it.
  - can use Fleming's LHR, bearing in mind electrons travel on the opposite direction to conventional current
- Since the resultant voltage across the semiconductor, V_{H}, is proportional to flux density, B, we can use the Hall effect in a transducer whose output voltage varies in response to a changing magnetic field.
- Characteristic curve of a Hall sensor is a very stretched 'S' — horizontal at saturation (minimum and maximum B) but linear in between. At When there is no significant magnetic field present then V_{H} is halfway between the saturation points.
- A Hall sensor can be used to detect the position and attitude of an object in three dimensions — requires attaching a magnet to the object. If the magnet is at an angle then this changes the component of B normal to the sensor, which changes the output Hall voltage.
- Hall sensors can also be used in tachometers to measure rotational speeds of wheels or machinery. Such tachometers can work at frequencies of 100 kHz or more which means they can measure very fast rotational speeds.
  - advantage that they do not make physical contact with the shaft so there are no frictional forces

15.2 Analogue and digital signals

15.2.1 Analogue to digital conversion

- When sampling an analogue signal we must do so at at least twice the highest information frequency so that information is not missed (Nyquist frequency)
- Number of bit sin each measurement gives the resolution
- Increasing sample rate & resolution will increase quality of conversion
- Quantisation is the number of available voltage values — when a sample is taken the snapshot is rounded or 'quantised' — the difference between the analogue and digital value is the 'approximation' or 'quantising' error.

15.2.2 Advantages & disadvantages of digital signals

- Advantages
  - high immunity to noise — maintains quality over long distances
  - if digital signal is affected by a high level of noise the original signal can be recovered
  - more information can be sent by digital signals than analogue signals in the same time using the same medium
  - transmitted signals can be interfaced readily with other digital systems such as computers
  - digital signals enable optical transmission through optical fibres with added advantages of security, freedom from interference, and lighter cabling
- Disadvantages
  - information is not as exact as that of an analogue signal because discrete values are used
  - systems for transmission & processing can be more complex to build than for analogue systems

Noise consists of random voltage fluctuations at different frequencies in the signal — due to interference from electronic processing systems as well as external sources.

Each time a signal is amplified the noise is too — eventually it may be impossible to communicate due to the noise.

$$\text{signal to noise ratio in dB} = 10 \log \frac{P_{\text{signal}}}{P_{\text{noise}}}$$

Noise can be removed from a digital signal thanks to the fact they consist of two discrete voltage levels. A device such as a Schmitt trigger could be used to recondition the noisy digital signal eg in a regenerator.

Sensors are devices that detect and respond to a certain type of analogue input. They are transducers — as well as detecting a physical quantity they convert such quantities into an electrical signal.

- sensitivity is the amount of change in output quantity with unit change in input quantity
- resolution — smallest change in physical property that can be measured as a ratio of the actual value measured
- response time is how long it takes to completely respond to a change in input

15.2.3 Pulse Code Modulation

1. sampling — reading values of analogue signal at equal time intervals at constant sampling rate
2. quantisation — assigning a discrete value from a range to each sample
3. encoding — representing the sampled values as a binary number

The resulting stream of binary numbers is a pulse code modulated signal.

$$\text{bit rate} = \text{number of quantisation bits} \times \text{sampling frequency}$$

15.3 Analogue signal processing

15.3.1 LC resonance filters

For this spec, need only to know of the parallel inductor—capacitor filter.

- inductor stores energy in the form of a magnetic field. Its action can be explained Faraday's & Lenz's Law: $\varepsilon_{\text{ind}} \propto -\frac{\mathrm{d}\phi}{\mathrm{d}t}$
  - unit is the Henry. 1 H when current changing at 1 A s^{-1} induces 1 V across coil. 1 H = 1 kg m^2 s^{-2} A^{-2}
- energy is constantly being exchanged between the magnetic field of the inductor and the electric field of the capacitor such that the LC network can be said to oscillate.

Comparison with mass—spring system

- with energy stored solely as electrostatic potential energy in the capacitor the LC network is similar to a stretched spring at maximum displacement (where E is stored as E_p in spring & mass is stationary)
- with energy solely in inductor's magnetic field can be compared to the spring at equilibrium: mass moves with max v and all energy is E_k.
- inductance can be compared to mass, and capacitance compared to the spring

If a driving force is applied with a frequency close or equal to a system's natural frequency large amplitude oscillations build up — resonance.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Energy response At the resonant frequency, f_0, energy stored in the circuit is at a maximum and the voltage across the LC circuit is at its peak. The voltage response curve is the same shape as energy.

Bandwidth, f_B, is given by the range of frequencies between the 50% energy (or power) points, or the points where voltage response is $\frac{1}{\sqrt{2}}$ of the maximum.

$$\text{Q factor} = \frac{f_0}{f_B}$$

High Q filters have sharper peaks while low-Q have shallower/gentler graphs.

15.3.2 The ideal operational amplifier

- The ideal op-amp has:
  - infinite open loop gain (A_{OL})
  - infinite input resistance (between V_+ & V_-)
  - no output resistance
  - maximum and minimum output voltages equal to the supply voltage
- in reality:
  - open loop gain of $\gtrapprox 10^5$
  - input resistance $\gtrapprox 10^7$ to 10^{12}
  - non-zero input resistance
  - maximum output V 1-2 V off V_S
- Ideally an op-amp would have constant gain regardless of frequency, but a graph of frequency response shows gain drops as frequency increases. For first 10 Hz or so there is flat frequency response, but it then falls off in a straight line (on a log-log graph)

$$\text{gain} \times \text{bandwidth} = \text{gain—bandwidth product (constant)}$$

For an op-amp, $V_{\text{out}} = A_{\text{OL}}(V_+ - V_-)$ — but thanks to the massive differential gain of the op-amp a very small difference between the inputs will send it into saturation. This means an op-amp can be used as a comparator — giving a high or low output depending on which input is higher ($V_+ \geq V_- \implies V_{\text{out}} \approx +V_S$, $V_- \geq V_+ \implies V_{\text{out}} \approx 0$ V or $-V_S$)

15.4 Controlling the op-amp

15.4.1 Inverting amp

Derivation of the inverting op-amp equation

- ‘virtual earth analysis’
  - V_+ at 0 V
    $\therefore V_{\text{out}} = A_{\text{OL}}(0 - V_-) = -A_{\text{OL}}V_-$
    $\implies V_- = -\frac{V_{\text{out}}}{A_{\text{OL}}}$
  - since A_{OL} is massive we can regard V_- as 0 V — virtual earth (so close to 0 V that it can be considered such)
    - A virtual earth point is not actually connected to 0 V, but because one op-amp input is connected to 0 and the difference between the op-amp inputs must be as small as possible due to negative feedback, the V.E.P. ends up being so close to 0 V it is considered 0 V.
- Assuming no current draw by input as infinite input resistance (ideal op-amp) $\therefore I_{\text{R}_{\text{in}}} = I_{\text{R}_{\text{f}}}$
  - So from $V = IR$:
    $$\frac{V_{\text{out}}}{R_{\text{f}}} = -\frac{V_{\text{in}}}{R_{\text{in}}} \text{ (negative sign as inverting input)} = A_{\text{CL}}$$
- This is now the ‘closed loop gain’ as a feedback loop exists from output to input. Negative feedback as it reduces the gain.

15.4.2 Non-inverting amplifier

$$\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_{\text{f}}}{R_1}$$

Has the advantage of very high input resistance as it goes directly into the op-amp input.

15.4.3 Summing amplifier

$$V_{\text{out}} = -R_{\text{f}}(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \ldots)$$

15.4.4 Difference amplifier

$$V_{\text{out}} = \frac{R_{\text{f}}}{R_1}(V_+ - V_-)$$

15.5 Digital signal processing

15.5.1 Combinational logic

Be aware of Boolean algebra & logic gates.

$$\bar{A}.B + A.\bar{B} = A \oplus B$$

15.5.2 Sequential logic

Binary counter D-type flip-flops can be connected together to make a counter that counts in binary. For n-bit counting, n flip-flops are needed.

Connecting D to $\bar{\text{Q}}$ makes the flip-flop 'toggle' with each rising-edge clock pulse divide-by-two counter.

At the rising edge of a clock pulse the signal at D is copied to Q.

Putting toggling flip-flops in series with the clock to the previous $\bar{\text{Q}}$ creates a binary up-counter. Q to clock would be a down-counter.

Modulo-n counter A modulo-n counter counts up to a chosen number. This is done by detecting the binary state of the number using external logic gates and sending the output of this to the reset pin.

Binary coded decimal counter A binary coded decimal counter is a case of the modulo-n that counts from 0 to 9 and then resets. Thus goes through a sequence of 10 states.

Johnson counter This is a 'shift register' — connect the D-type flip-flops together so the Q of one feeds the D of the next, when clocked at simultaneously the data is then shifted along the flip-flops sequentially.

Specifically in a 'Johnson' counter, the $\bar{\text{Q}}$ output of the last flip-flop is fed back to the first D. The data pattern contained within the register will recirculate as long as the clock pulses are applied. An n-stage Johnson counter will give a count sequence of length $2n$ before the pattern is repeated.

The most significant binary bit will be the furthest right flip-flop (changes least)

15.5.3 Astable

The astable can be used as an oscillator to provide a clock pulse. Its frequency and therefore time period are adjusted using an external RC network.

It can be difficult to obtain an even mark:space ratio, but using a divide-by-two counter will ensure this where needed.

15.6 Data communication systems

15.6.1 Transmission media

- Transmission-path media include metal wire, optical fibre, electromagnetic wave (free space)
- Radio waves can travel through ground wave, or by sky wave where signals are refracted through the ionosphere and 'bounce' around the Earth. Longer wavelengths can also diffract around the Earth's surface.
- Uplink and downlink channels need to be on separate frequencies so that receives are not desensed.
    - Satellite uplink & downlink use microwave frequencies (those in the GHz range). Band C (3.4 to 7 GHz) hosts fixed telephony, broadcast radio and business networks, while encrypted government & military communications are sent on band X (7 to 8.4 GHz) and high bit-rate transmission such as television and videoconferencing is found on the Ku and Ka bands (10.7 to 31 GHz).
    - The downlink is on a lower frequency than the uplink, which helps reduce the attenuation of the relatively weak downlink — the higher the frequency, the greater the attenuation of a radio wave in the atmosphere.

15.6.2 Multiplexing

Multiplexing is a way of transmitting multiple signals along a medium so as to maximise the available bandwidth. In time division multiplexing, multiple signals are sent along the same medium (on the same frequency, where applicable) by dividing them in the time domain. Each signal is sampled periodically and converted to a stream of digital bits, which are transmitted in their own specific time slot. The receiver then combines the data and decodes it to obtain the original signal.

15.6.3 Modulation

In AM, the frequency of the carrier is kept constant but its amplitude is varied in phase with the amplitude of the information signal. The frequency of the information signal is represented by how often these variations in amplitude occur.

$$\text{modulation depth} = \frac{\text{information amplitude}}{\text{carrier amplitude}} \times 100$$

The target modulation depth is 80%. Over 100% is 'overmodulated', and if modulation depth is too small the signal to noise ratio is reduced and quality suffers.

When two frequencies are mixed together, we get 'side' frequencies at the sum and difference of these signals.

$$\text{AM bandwidth} = 2f_M$$

AM circuitry is simpler and therefore cheaper than FM, but AM is more susceptible to noise, as noise adds unwanted energy to a signal — affecting the amplitude, while no information is contained in the amplitude of an FM signal.

$$\text{FM bandwidth} = 2(f_M + \Delta f)$$